# Automating the derivation of the Bondi mass loss formula for modified theories of gravity

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## Motivation

The Bondi-Sachs formalism describes the flow of energy leaving an isolated gravitational system. This could be used as a observable signature from alternative theories of gravity.

Additionally, an important prediction of the formalism is the existence of a *gravitational wave memory effect*.

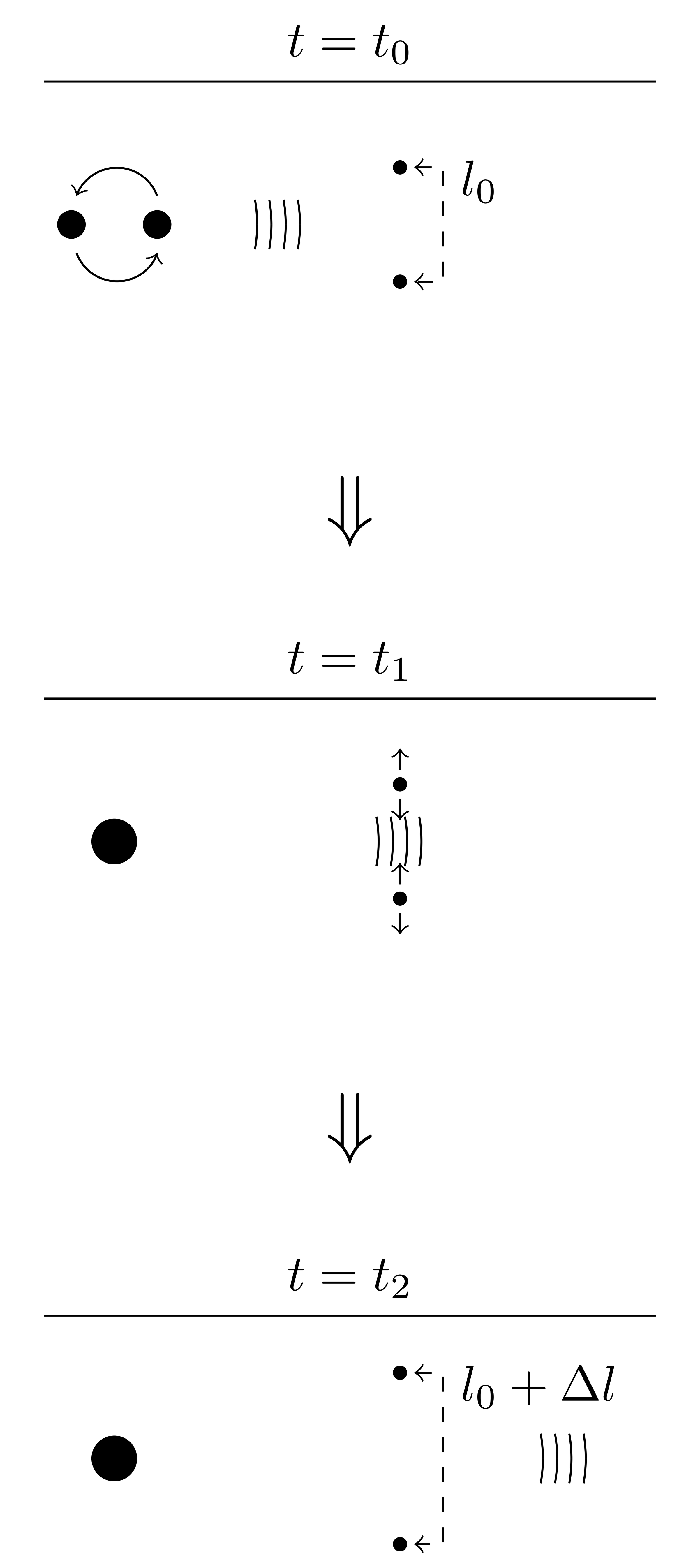



Illustration of the memory effect as burst of gravitational waves goes through space between free falling particles.

At conformal infinity, it also introduces the asymptotic symmetries known as the BMS group. This topic has been of great interest in the search for a quantum theory of gravity [2].

We intend to extend this framework to more general families of theories and, in the process, write a computer algebra package to automate these steps.

## Theoretical Background[3]

The formalism is based on the Bondi-Sachs metric

$$g_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b = -\frac{V}{r}e^{2\beta}\,\mathrm{d}u^2 - 2e^{2\beta}\,\mathrm{d}u\,\mathrm{d}r + r^2 h_{AB}(\mathrm{d}\sigma^A - U^A\,\mathrm{d}u)(\mathrm{d}\sigma^B - U^B\,\mathrm{d}u)$$

It contains six metric quantities and the coordinates are chosen such that only the $r$ coordinate varies along the light cones.

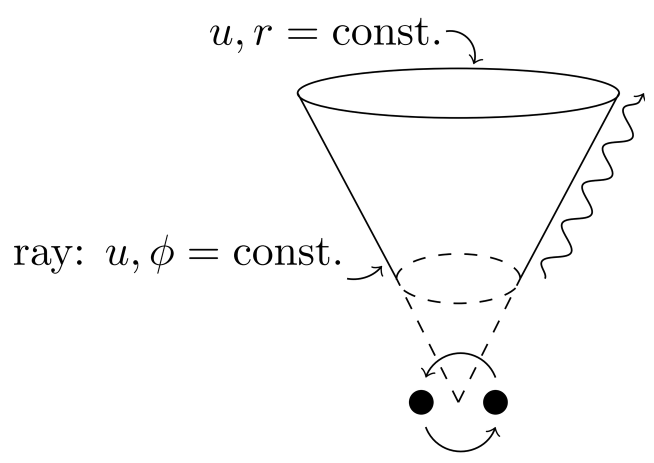



Illustration of the coordinate system with $\theta$ excluded.

We define the tensor $E_{ab}$ representing Einstein's field equations as

$$E_{ab} = R_{ab} - \frac{1}{2}g_{ab}R - 8\pi T_{ab}$$

and designate

$$E^u{}_a = 0, \quad E_{AB} - \frac{1}{2}g_{AB}g^{CD}E_{CD} = 0$$

as the main equations.

The key insight is to notice that the main equations imply the supplementary conditions

$$\partial_r(r^2 e^{2\beta} E^r{}_u) = 0, \quad \partial_r(r^2 e^{2\beta} E^r{}_A) = 0$$

Thus, if $E^r{}_u$ and $E^r{}_A$ are known for some $r = r_0$ or $r \to \infty$, then they are determined for every other value $r$.

To solve the main equations, we use an ansatz based on a power series in $r^{-1}$ and proceed order by order. This introduces the functions

$$c_{AB} = \lim_{r\to\infty} r(h_{AB} - q_{AB})$$

and

$$M = -\frac{1}{2}\lim_{r\to\infty}\,[V - r]$$

From the former, we define the *news tensor* as

$$N_{AB} = \frac{1}{2}\partial_u c_{AB}$$

which is connected to the mass flow out of the system through the first of the supplementary conditions. This yields the Bondi mass loss formula

$$\partial_u M = \frac{1}{2}\left(\eth_A\eth_B N^{AB} - N_{AB}N^{AB}\right)$$

Note that this result is, in principle, not limited to GR. Such formulae may be found by a similar procedure for any alternative theory of gravity that introduces an effective stress-energy tensor.

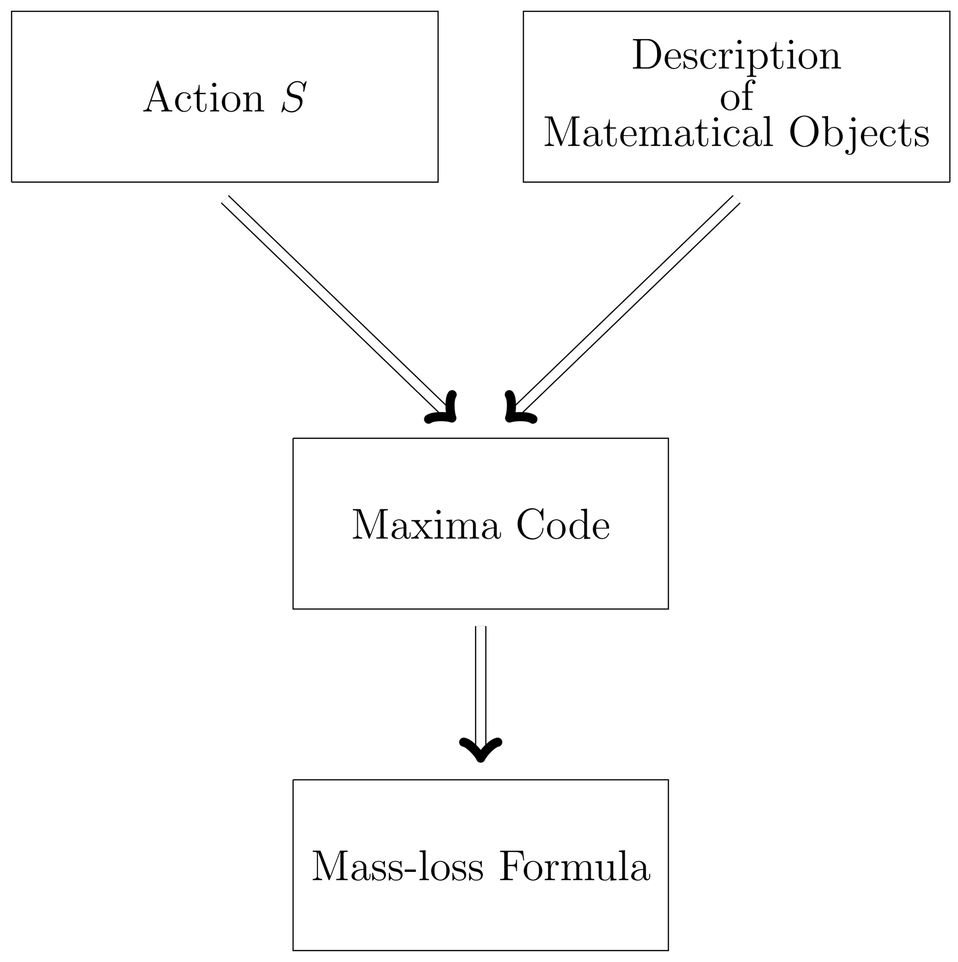
The importance of this result is summarized in the statement of the original 1962 paper [1].

> *The mass of a system is constant if and only if there is no news. If there is news, the mass decreases monotonically as long as the news continues.*

## Objective

As explained above, the procedure to calculate the Bondi mass loss formula for different theories of gravity is clear up to specifics, like the definition of asymptotic flatness.

Our current work is to develop a computer algebra package to automate the derivation of mass loss formulae starting from an action.




At this stage, we temporarily restrict ourselves to axially symmetric systems. We are working on mass loss formulae for scalar-tensor theories from the Lagrangian density

$$\mathcal{L}_{\mathrm{SC}} = \frac{1}{2\kappa^2}\left[A(\Psi)R - B(\Psi)\,\partial^a\Psi\,\partial_a\Psi - V(\Psi)\right]\sqrt{|\det g|}$$

and Einstein-Gauss-Bonnet-Dilaton theory, given by the Lagrangian density

$$\mathcal{L}_{\mathrm{EGBd}} = \frac{1}{2\kappa^2}\left[R - \frac{1}{2}\partial^a\Phi\,\partial_a\Phi + \frac{1}{4}\alpha e^{\gamma\Phi}R_{\mathrm{GB}}\right]\sqrt{|\det g|}$$

Further work will extend the software to the general case presented in the theoretical background section.

## Additional Information

Revisions of this poster and the source code of the maxima script used to check the original paper of Bondi et al[1] can be found at:

`https://thomaslabs.org/research/tartu2026.html`

When done, the source code for more general calculations will also be made publicly available.

## References

[1] H. Bondi et al. In: (1962). DOI: 10.1098/rspa.1962.0161.
[2] Ciambelli L. et al. In: (2023). DOI: 10.48550/arXiv.2307.08460.
[3] Thomas Mädler et al. In: (2016). DOI: 10.4249/scholarpedia.33528.