# Conformal and disformal transformations using Mathematica

A case study in xAct

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- Code available to download (see last slide)



(Based on Appendix D of Wald 1984)

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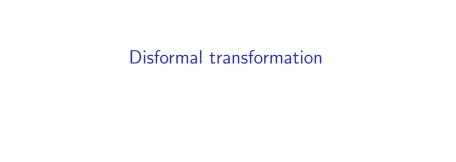
$$D_a\omega_b = \nabla_a\omega_b - C_{ab}^c\omega_c,$$

where

$$C_{ab}^{c}=2\delta_{\phantom{c}(a}^{c}\nabla_{b)}\ln\Omega-g_{ab}g^{cd}\nabla_{d}\ln\Omega$$

Finally, the curvature  $\tilde{R}_{abc}^{\phantom{abc}d}$ , associated with  $D_a$  and the curvature,  $R_{abc}^{\phantom{abc}d}$ , associated with  $\nabla_a$  is given by:

$$\begin{split} \tilde{R}_{abc}{}^{d} &= R_{abc}{}^{d} + 2 \delta^{d}{}_{[a} \nabla_{b]} \nabla_{c} \ln \Omega \\ &- 2 g^{de} g_{c[a} \nabla_{b]} \nabla_{e} \ln \Omega \\ &+ 2 \left( \nabla_{[a} \ln \Omega \right) \delta^{d}{}_{b]} \nabla_{c} \ln \Omega \\ &- 2 \left( \nabla_{[a} \ln \Omega \right) g_{b]c} g^{df} \nabla_{f} \ln \Omega \\ &- 2 g_{c[a} \delta^{d}{}_{b]} g^{ef} \left( \nabla_{e} \ln \Omega \right) \nabla_{f} \ln \Omega \end{split}$$



(Based on Alinea 2020)

We introduce a new metric  $h_{ab}$  defined by:

$$h_{ab} = A(\phi)g_{ab} + B(\phi)(\nabla_a\phi)\nabla_b\phi$$
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where the inverse can be derived using the *Sherman-Morrison* formula:

$$(M + uv^T)^{-1} = M^{-1} - \frac{M^{-1}uv^TM^{-1}}{1 + v^TM^{-1}u}$$

The transformed Christoffel symbols are given by:

$$\hat{\Gamma}^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} + C^{\alpha}_{\mu\nu} 
= \Gamma^{\alpha}_{\mu\nu} + \frac{A'}{2A} \left( \delta^{\alpha}_{\mu} \nabla_{\nu} \phi + \delta^{\alpha}_{\nu} \nabla_{\mu} \phi \right) - \frac{A'}{2(A - 2BX)} g_{\mu\nu} \nabla^{\alpha} \phi 
+ \frac{AB' - 2A'B}{2A(A - 2BX)} (\nabla^{\alpha} \phi) (\nabla_{\mu} \phi) \nabla_{\nu} \phi 
+ \frac{B}{A - 2BX} (\nabla^{\alpha} \phi) \nabla_{\mu} \nabla_{\nu} \phi$$

where

$$X=-rac{1}{2}{f g}^{\mu
u}(
abla_{\mu}\phi)
abla_{
u}\phi$$

Finally, the transformed Riemann curvature tensor is given by:

$$\begin{split} \hat{R}^{\alpha}_{\ \mu\beta\nu} &= \qquad -\partial_{\nu}\hat{\Gamma}^{\alpha}_{\mu\beta} + \partial_{\beta}\hat{\Gamma}^{\alpha}_{\mu\nu} - \hat{\Gamma}^{\rho}_{\mu\beta}\hat{\Gamma}^{\alpha}_{\rho\nu} + \hat{\Gamma}^{\rho}_{\mu\nu}\hat{\Gamma}^{\alpha}_{\rho\beta} \\ &= \qquad R^{\alpha}_{\ \mu\beta\nu} - \partial_{\nu}C^{\alpha}_{\mu\beta} + \partial_{\beta}C^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\rho\nu}C^{\rho}_{\mu\beta} - \Gamma^{\rho}_{\mu\beta}C^{\alpha}_{\rho\nu} \\ &\qquad C^{\rho}_{\mu\beta}C^{\alpha}_{\rho\nu} + \Gamma^{\alpha}_{\rho\beta}C^{\rho}_{\mu\nu} + \Gamma^{\rho}_{\mu\nu}C^{\alpha}_{\rho\beta} + C^{\rho}_{\mu\nu}C^{\alpha}_{\rho\beta} \end{split}$$

#### Sources

- Alinea, Allan L. (2020). On the Disformal Transformation of the Einstein-Hilbert Action. arXiv: 2010.00956 [gr-qc].
- Martín-García, José M. (2025). xAct: Efficient tensor computer algebra for the Wolfram Language. URL:
  - https://josmar493.dreamhosters.com/.
- Wald, R.M. (1984). *General Relativity*. University of Chicago Press. ISBN: 9780226870373.

Mathematica notebooks and these slides are available on my website

https://thomaslabs.org/research/xact-example.html

